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Dynamics of (SUSY) AdS space isometry breaking

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Abstract

Actions governing the dynamics of the Nambu–Goldstone modes resulting from the spontaneous breaking of the $SO(4,2)$ and $SU(2,2|1)$ isometries of five-dimensional anti-de Sitter space (AdS_5) and SUSY $AdS_5 \times S_1$ spaces, respectively, due to a restriction of the motion to embedded four-dimensional AdS_4 space and four-dimensional Minkowski space (M_4) probe branes are presented. The dilatonic Nambu–Goldstone mode governing the motion of the M_4 space probe brane into the covolume of the SUSY $AdS_5 \times S_1$ space is found to be unstable. No such instability appears in the other cases. Gauging these symmetries leads to an Einstein–Hilbert action containing, in addition to the gravitational vierbein, a massive Abelian vector field coupled to gravity.

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Conformal and superconformal invariance play a pivotal role in many currently investigated theoretical models. A major advance which has further elucidated these studies was the conjectured correspondence between certain (super) conformal field theories and theories formulated on anti-de Sitter (AdS) and supersymmetric (SUSY) AdS spaces [1]. Here, I present various dynamical consequences for field theories on AdS spaces and their supersymmetric extensions which arise due to the spontaneous breakdown of some of their spacetime symmetries when the motion is restricted to lower dimensional probe branes.

A background AdS_5 space is characterized by a constant Ricci scalar curvature, $R = -20m^2$, and has the isometry group $SO(4,2)$ whose generators, $M^{MN} = -M^{NM}$, $M, N = 0, 1, 2, 3, 4, 5$, satisfy the algebra:

$$[M_{MN}, M_{LR}] = \mathrm{i}(\hat{\eta}_{ML}M_{NR} - \hat{\eta}_{MR}M_{NL} - \hat{\eta}_{NL}M_{MR} + \hat{\eta}_{NR}M_{NL}) \tag{1}$$

where $\hat{\eta}_{MN}$ is a diagonal metric tensor with signature $(-1, +1, +1, +1, +1, -1)$. For embedding AdS_4 space, it is useful to parametrize the AdS_5 space with the coordinates $\rho, x^\mu, \mu = 0, 1, 2, 3$, so that the AdS_5 space invariant interval takes the form

$$\mathrm{d}s^2 = \mathrm{e}^{2A(\rho)}\eta_{\lambda\mu}\bar{e}^\sigma_\nu(x)\bar{e}^\lambda_\sigma(x)\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu + (\mathrm{d}\rho)^2 \tag{2}$$

where $A(\rho) = \ell n[\cosh(m\rho)]$ is a warp factor and $\bar{e}^\nu_\mu(x) = \frac{\sinh(\sqrt{m^2x^2})}{\sqrt{m^2x^2}}P^\nu_{\perp\mu}(x) + P^\nu_{\|\mu}(x)$ is the AdS_4 vierbein. Here, $P^\nu_{\perp\mu}(x) = \eta^\nu_\mu - \frac{x_\mu x^\nu}{x^2}$ and $P^\nu_{\|\mu}(x) = \frac{x_\mu x^\nu}{x^2}$ are transverse and longitudinal projectors, respectively, while $\eta^{\mu\nu}$ is the 4D Minkowski space metric tensor with signature $(-1, 1, 1, 1)$. The AdS$_4$ invariant interval is obtained from the AdS$_5$ interval by setting $\rho = 0$ which in turn gives the orientation of AdS$_4$ brane in AdS$_5$ space. Such an embedding of AdS$_4$ space spontaneously breaks the isometry group of the AdS$_5$ space from $SO(4, 2)$ to $SO(3, 2)$. Introducing the (pseudo-) translation generators defined as $P_\mu = mM_{5\mu}$, $D = mM_{54}$, the broken generators are then identified as D and $M_{4\mu}$.

A model-independent way of encapsulating the long wavelength dynamical constraints imposed by this spontaneous symmetry breakdown is to realize the $SO(4, 2)$ isometry nonlinearly on the Nambu–Goldstone boson fields ϕ and v^μ associated with the broken symmetry generators D and $M_{4\mu}$, respectively. Using coset methods [2], the AdS$_5$ vierbein factorizes [3] as $e_\mu^\nu = \bar{e}_\mu^\lambda N_\lambda^\nu$ where $\bar{e}_\mu^\nu$ is the AdS$_4$ vierbein and $N_\lambda^\nu = \cosh(m\phi)\big\{\big[P_{\perp\lambda}^\nu(v) + \cos(\sqrt{v^2})P_{\|\lambda}^\nu(v)\big] + D_\lambda\phi\frac{\sin(\sqrt{v^2})}{\sqrt{v^2}}v^\nu\big\}$ with $D_\mu = \frac{1}{\cosh(m\phi)}\bar{e}_\mu^{-1\nu}\partial_\nu$ being the AdS$_4$ covariant derivative. The resultant $SO(4, 2)$ invariant action is $S = -\sigma\int \mathrm{d}^4x \det e$ with σ being the AdS$_4$ brane tension. Since this action is independent of $\partial_\mu v_\nu$, the Nambu–Goldstone field v_μ is non-dynamical [4] and it can be eliminated using its field equation $v^\mu\frac{\tan(\sqrt{v^2})}{\sqrt{v^2}} = D^\mu\phi$ so that the action can be recast as [3]

$$S = -\sigma\int \mathrm{d}^4x \det\bar{e}\cosh^4(m\phi)\sqrt{1 + D_\mu\phi D^\mu\phi}. \tag{3}$$

This Nambu–Goldstone field action contains a mass term, $m_\phi^2 = 4m^2$, along with non-derivative interactions and constitutes an AdS generalization of Nambu–Goto action [5]. Using the factorized AdS$_5$ vierbein, along with v field equation, the invariant interval for AdS$_5$ space takes the form

$$\mathrm{d}s^2 = \mathrm{e}^{2A(\phi)}\eta_{\lambda\mu}\bar{e}_\nu^\sigma(x)\bar{e}_\sigma^\lambda(x)\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu + (\mathrm{d}\phi(x))^2 \tag{4}$$

with $A(\phi) = \ell n[\cosh(m\phi)]$. This has the same structure as the invariant interval of AdS$_5$ space (equation (2)) obtained previously after the identification of $\phi(x)$ with the covolume coordinate ρ. As such, $\phi(x)$ describes the motion of AdS$_4$ brane into remainder of AdS$_5$ space.

Next, we embed a four-dimensional Minkowski space, M_4, probe brane into AdS$_5$ space [6]. For such an embedding, it proves convenient to introduce a different set of AdS$_5$ coordinates x^μ, x^4 so that the AdS$_5$ invariant interval is

$$\mathrm{d}s^2 = \mathrm{e}^{2mx_4}\,\mathrm{d}x^\mu\eta_{\mu\nu}\,\mathrm{d}x^\nu + (\mathrm{d}x_4)^2 \tag{5}$$

which reduces to the M_4 space invariant interval at $x_4 = 0$. Thus, inserting a Minkowski space probe brane at $x_4 = 0$, the broken generators are identified as $D, M^{4\mu}$ and the $SO(4, 2)$ isometry can be nonlinearly realized on the Nambu–Goldstone bosons, the dilaton, ϕ, and v^μ associated with these broken symmetry generators. The coset space construction allows the extraction of the AdS$_5$ vierbein as $e_\mu^\nu = \mathrm{e}^\phi\big[P_{\perp\mu}^\nu(v) + P_{\|\mu}^\nu(v)\cos(\sqrt{m^2v^2})\big] - \partial_\mu\phi v^\nu\frac{\sin(\sqrt{m^2v^2})}{\sqrt{m^2v^2}}$. Once again, v^μ is not independent dynamical degree of freedom. Eliminating it using its field equation $v^\mu\frac{\tan(\sqrt{v^2})}{\sqrt{v^2}} = -\mathrm{e}^\phi\partial^\mu\phi$ yields the invariant action term $-\sigma\int \mathrm{d}^4x\,\mathrm{e}^{4\phi}\sqrt{1 + \frac{1}{m^2}\mathrm{e}^{-2\phi}\partial_\mu\phi\eta^{\mu\nu}\partial_\nu\phi}$ while the invariant interval can be written as

$$\mathrm{d}s^2 = \mathrm{e}^{2\phi}\,\mathrm{d}x^\mu\eta_{\mu\nu}\,\mathrm{d}x^\nu + \frac{1}{m^2}(\mathrm{d}\phi)^2. \tag{6}$$

This has the same form as AdS$_5$ invariant interval of equation (5) after the identification of $\phi \Leftrightarrow \frac{1}{m}x_4$. Thus, the dilaton dynamics describes the motion of brane into the covolume of AdS$_5$ space. In the above, a particular combination for the broken generators was chosen. An alternate, equally valid, choice is D and $K^\mu = \frac{1}{m^2}P^\mu - \frac{1}{m}M^{4\mu}$. This, in turn, leads to the four-dimensional conformal algebra. Moreover, since the generators K^μ and $M^{4\mu}$ differ only by the unbroken translation generator P^μ, the action is also invariant under four-dimensional conformal transformations. Since $\mathrm{e}^{4\phi}$ transforms as total divergence under conformal transformations, the invariant term $\int \mathrm{d}^4x\, \mathrm{e}^{4\phi}$ can be subtracted producing the $SO(4,2)$ invariant action [7, 8]

$$S = -\sigma \int \mathrm{d}^4x\, \mathrm{e}^{4\phi}\left[\sqrt{1 + \frac{1}{m^2}\,\mathrm{e}^{-2\phi}\partial_\mu\phi\partial^\mu\phi} - 1\right] \tag{7}$$

which is defined so as to have zero vacuum energy.

Now consider embedding M_4 and AdS_4 branes in SUSY $\mathrm{AdS}_5 \times S_1$ space [9, 10]. The supersymmetric $\mathrm{AdS}_5 \times S_1$ isometry algebra, $SU(2,2|1)$, includes the generators $M^{\mu\nu}, P^\mu, M^{4\mu}, D$ of the $SO(4,2)$ isometry algebra, the SUSY fermionic charges $Q_\alpha, \bar{Q}_{\dot\alpha}, S_\alpha, \bar{S}_{\dot\alpha}$ and the R charge which is the generator of the $U(1)$ isometry of S_1. Embedding an M_4 probe brane at $x^4 = 0$ breaks the spacetime symmetries generated by P_4 and $M_{4\mu}$, as well as all the supersymmetries and the R symmetry. This $SU(2,2|1)$ isometry algebra of the super-$\mathrm{AdS}_5 \times S_1$ space can be nonlinearly realized on the Nambu–Goldstone modes of the broken symmetries [6]. These are the dilaton, ϕ, and v^μ associated with D and $M_{4\mu}$, respectively, the Goldstinos $\lambda_\alpha, \bar\lambda_{\dot\alpha}$ and $\lambda_{S\alpha}, \bar\lambda_{S\dot\alpha}$ of the spontaneously broken supersymmetries, $Q_\alpha, \bar{Q}_{\dot\alpha}, S_\alpha, \bar{S}_{\dot\alpha}$, and the R-axion a. The Nambu–Goldstone bosonic modes v^μ and the Goldstinos $\lambda_{S\alpha}, \bar\lambda_{S\dot\alpha}$ are not independent dynamical degrees of freedom [4] but rather are given in terms of the dilaton and Goldstinos $\lambda_\alpha, \bar\lambda_{\dot\alpha}$ as $v^\mu = \partial_\mu\phi + \cdots$, $\lambda_{S\alpha} = (\sigma^\mu\partial_\mu\bar\lambda)_\alpha + \cdots$ and $\bar\lambda_{\bar{S}\dot\alpha} = (\partial_\mu\lambda\sigma^\mu)_{\dot\alpha} + \cdots$. After elimination of the non-dynamical Nambu–Goldstone modes, the resultant invariant action is

$$S = -\sigma \int \mathrm{d}^4x\, \mathrm{e}^{4\phi} \det A \sqrt{1 + \frac{\mathrm{e}^{-2\phi}}{m^2}\mathcal{D}_\mu\phi\mathcal{D}^\mu\phi}\,[1 + \mathrm{e}^{-2\phi}\mathcal{D}_\mu a\mathcal{D}^\mu a][1 + B] \tag{8}$$

where $A_\mu^\nu = \eta_\mu^\nu + \mathrm{i}(\lambda\overleftrightarrow{\partial}_\mu\sigma^\nu\bar\lambda)$ is the Akulov–Volkov vierbein [11], $\mathcal{D}_\mu = A_\mu^{-1\nu}\partial_\nu$ is the SUSY covariant derivative and B is a somewhat lengthy sum of terms all of which are least bilinear in the Goldstino fields and contain at least two derivatives [6]. The action is an invariant synthesis of Akulov–Volkov and Nambu–Goto actions. Note that the pure dilatonic part of the action (obtained by setting the Goldstinos and R-axion to zero so that $A_\mu^\nu = \delta_\mu^\nu$ and $B = 0$) reproduces the previous action of the Minkowski space M_4 probe brane in AdS_5 without SUSY. As such, the dilaton ϕ describes the motion of the probe brane into the rest of the AdS_5 space. However, in this case, because of the spontaneous breakdown of the complete SUSY, there is no invariant that can be added to the action to cancel the vacuum energy such as one was able to achieve in the non-supersymmetric Minkowski space probe brane case (cf equation (7)). It follows that the dilaton feels an $\mathrm{e}^{4\phi}$ potential which, in turn, contains a destabilizing term linear in ϕ driving the dilaton field to $\phi \to -\infty$. Since the dilaton describes the motion of the probe Minkowski M_4 brane into the remainder of AdS_5 space, it follows that the SUSY AdS_5 space cannot sustain the Minkowski space brane.

The alternate combination of broken generators D and $K_\mu = \frac{1}{m^2}(P_\mu - 2mM_{4\mu})$ can also be defined. This leads to the 4D superconformal algebra. The spontaneously broken symmetries are R, dilatations (D), special conformal (K^μ), SUSY ($Q_\alpha, \bar{Q}_{\dot\alpha}$) and SUSY conformal ($S_\alpha, \bar{S}_{\dot\alpha}$). Since the generators K^μ and $M^{\mu 4}$ differ only by unbroken translation generator P^μ, the action (8) is invariant under superconformal transformations. Once again the potential for the dilaton ϕ is unstable and there is an incompatibility of simultaneous nonlinear realizations of SUSY and scale symmetry in four-dimensional Minkowski space [12]. Alternatively expressed, the spectrum of four-dimensional Minkowski space cannot include both the Goldstino and the dilaton as Nambu–Goldstone modes. Note that the origin of this unusual behaviour is not simply a consequence of the introduction of a scale due the spontaneously broken SUSY. It has been shown that there is no incompatibility in securing simultaneous nonlinear realization of spontaneously broken scale and chiral symmetries [13] where a scale is also introduced. In that case, the spectrum of the effective Lagrangian admits both pions and a dilaton.

On the other hand, the invariant action for the dilaton ϕ and Goldstinos obtained by embedding an AdS_4 probe brane in SUSY $\mathrm{AdS}_5 \times S_1$ space has, in addition to other modifications, an overall prefactor of $\cosh^4(m\phi)$ instead of $\mathrm{e}^{4\phi}$. Thus, in this case, there is no destabilizing linear in ϕ term. Consequently an AdS_4 brane can be embedded in SUSY $\mathrm{AdS}_5 \times S_1$ space and the spectrum can admit both a massive dilaton and massive Goldstinos.

Thus far, we have focused on a fixed background AdS_5 space and the actions constructed are invariant under a nonlinear realization of the global isometry group $SO(4,2)$. In order to describe the dynamics of an oscillating brane embedded in curved space, we need to have invariance under local $SO(4,2)$ transformations and additional gauge fields including dynamical gravity must be introduced. The dynamics of the brane embedded in curved space is then described by a brane localized massless graviton [14, 15] represented by a dynamical metric tensor $g_{\mu\nu}$ and a vector field $A_\mu(x)$. As a consequence of the Higgs mechanism, the vector field is massive [16]. The action for these fields is once again derived in a model-independent manner using coset methods. Isolating the physical degrees of freedom by working in unitary gauge defined by setting $\phi = 0$ and $v^a = 0$, the action takes the form [17]

$$S = \int \mathrm{d}^4x\sqrt{-\det g}\left\{-\frac{1}{16\pi G_N}(2\Lambda + R) - \frac{1}{4}F_{\mu\nu}g^{\mu\rho}g^{\nu\sigma}F_{\rho\sigma} + \frac{1}{2}A_\mu[(M^2 + c_1 R)g^{\mu\nu} + c_2 R^{\mu\nu}]A_\nu\right\} \tag{9}$$

where Λ is the cosmological constant, G_N is Newton's constant, $R^{\mu\nu}(R)$ is the full (background plus dynamical) Ricci tensor (scalar), while $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the Abelian field strength and c_1, c_2 are constants. This is recognized as the action of a massive Proca field A_μ with independent mass parameter M interacting with either AdS_4 or M_4 Einstein gravity. When coupled to the standard model, this Abelian vector field transforms analogously to the weak hypercharge gauge field and thus will lead to a Z' boson in the spectrum. Note that since the vector mass M is an independent parameter, it is nonzero even in the flat space limit ($m = 0$) and consequently such a massive Abelian Proca field also appears when an M_4 brane probe is inserted in M_5 space in a locally invariant manner.

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